Optimality test (UV method)

Step-1 :- First find a basic feasible sol^n of the given transportation problem by any one of the methods (specially VAM). For non-degenerate problem there will be $(m+n-1)$ independent positive allocations.

Step-2 Determine a set of $(m+n)$ numbers u_i and v_j : $(i = 1, 2, \ldots m \; ; \; j = 1, 2, \ldots n)$ such that for all occupied (i, j) cells $c_{ij} = u_i + v_j$.
In practice to find u_i and v_j put any one

of them equal to zero and other u_i's & v_j's can be ~~put out~~ found out. Generally that u_i ~~and~~ or v_j for which the corresponding row or column contains maximum no. of occupied cells in put to zero.

Step-3 :- Calculate the cell evaluation Δ_{ij} for each unoccupied (i,j) cells by the formula $\Delta_{ij} = u_i + v_j - c_{ij}$ and put them in the upper right corners of the corresponding unoccupied cells.

~~Step-4~~ :- Case-(i) :- In all $\Delta_{ij} < 0$ then the solution obtained is optimal and unique.

Case-(ii) :- If all $\Delta_{ij} \leq 0$ with ~~the~~ a least one $\Delta_{ij} = 0$ then the solution optimal but not unique.

Case-(iii) :- If at least one $\Delta_{ij} > 0$ then the solution optimal but not unique, and we go ~~to step-4~~.

Step-4 :-

Find the optimal solution of the following T.P

	D_1	D_2	D_3	D_4	
O_1	5	4	8	14	15
O_2	2	9	9	6	4
O_3	6	11	7	13	8
	9	7	5	6	

<u>solution:</u> Since $\sum a_i = \sum b_j = 27$ So it is balanced T.P.

An initial basic feasible solution is obtained by VAM is shown.

	D_1	D_2	D_3	D_4				
O_1	$\boxed{8}$ 5	$\boxed{7}$ 4	8	14	15(1)	15(1)	8(1)	
O_2	2	9	9	$\boxed{4}$ 6	4(4)			
O_3	$\boxed{1}$ 6	11	$\boxed{5}$ 7	$\boxed{2}$ 13	8(1)	8(1)	8(1)	8
	9 (3)	7 (5)	5 (1)	6 (7)				
	9 (1)	7 (7)	5 (1)	2 (1)				
	9 (1)		5 (1)	2 (1)				
	1		5	2				

No. of basic cells = 6 (m+n-1) so it is a non-degenerate T.P. We check optimality condition by UV method.

Now the values of u_i, v_j and cell evaluations are done as follows —

calculations of u_i and v_j

Basic variables	(u,v) eqn	solutions
x_{11}	$u_1 + v_1 = 5$	$v_1 = 6 \Rightarrow u_1 = -1$
x_{12}	$u_1 + v_2 = 4$	$u_1 = -1 \Rightarrow v_2 = 5$
x_{24}	$u_2 + v_4 = 6$	$v_4 = 13 \Rightarrow u_2 = -7$
x_{31}	$u_3 + v_1 = 6$	$u_3 = 0 \Rightarrow v_1 = 6$
x_{33}	$u_3 + v_3 = 7$	$u_3 = 0 \Rightarrow v_3 = 7$
x_{34}	$u_3 + v_4 = 13$	$u_3 = 0 \Rightarrow v_4 = 13$

$\therefore u_1 = -1;\ u_2 = -7;\ u_3 = 0;\ v_1 = 6;\ v_2 = 5$
$v_3 = 7;\ v_4 = 13.$

Cell evaluations of non basic cells

For non basic variable $\Delta_{ij} = u_i + v_j - c_{ij}$

Here non basic variables are $x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{32}$.

non basic variables	
x_{13}	$\Delta_{13} = u_1 + v_3 - c_{13} = -1 + 7 - 6 = 0$
x_{14}	$\Delta_{14} = u_1 + v_4 - c_{14} = -1 + 13 - 14 = -2$
x_{21}	$\Delta_{21} = u_2 + v_1 - c_{21} = -7 + 6 - 2 = -3$
x_{22}	$\Delta_{22} = u_2 + v_2 - c_{22} = -7 + 5 - 9 = -11.$

x_{23} $\quad A_{23} = u_2 + V_3 - C_{23} = -7 + 7 - 9 = -9$

x_{32} $\quad A_{32} = u_3 + V_2 - C_{32} = 0 + 5 - 11 = -6$

Since all non basic cell evaluations are negetive or zero, so the optimal soln is reached. The solution is

$x_{11} = 8$; $x_{12} = 7$; $x_{24} = 4$; $x_{37} = 1$; $x_{33} = 5$
$x_{34} = 2$

$$\text{cost} = (8 \times 5) + (7 \times 4) + (4 \times 6) + (1 \times 6) + (5 \times 7) + (2 \times 13) \text{ unit}$$

$$= 159 \text{ unit.}$$

H.W Find the optimal solution of the following problem

	D_1	D_2	D_3	D_4	
O_1	21	16	25	13	11
O_2	17	18	14	23	13
O_3	32	27	18	41	19
	6	10	12	15	